MENIIT
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JEE Advanced : Paper-I (2017)

IMPORTANT INSTRUCTIONS

GENERAL

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at 9:00 am, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). TheCandidate's Sheet is a carbonless copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. Do not use the ORS for rough work.
14. Write your name, roll number and code of the examination center, and sign with pen is the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

15. Use a BLACK BALL POINT PEN to darken the bubbles on the ORS.
16. Darken the bubble COMPLETELY.
17. The correct way of darkening a bubble is as:
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un-darken" a darkened bubble. Please

PART A: PHYSICS

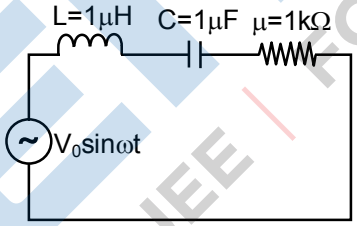
SECTION 1 (Maximum Marks : 28)

- This section contains SEVEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

1. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(A)At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(B)The frequency at which the current will be in phase with the voltage is independent of R

(C)The current will be in phase with the voltage if $\omega = 10^4$ rad. s^{-1}

(D) At $\omega >> 10^6$ rad. s^{-1}, the circuit behaves like a capacitor

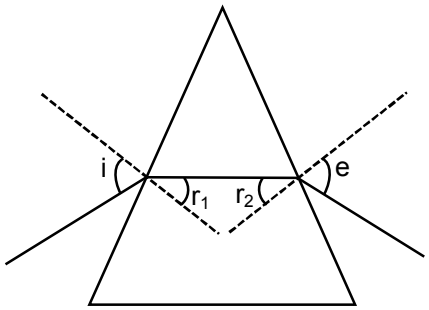
Ans. AB

Sol. $\tan\phi = \frac{X_L - X_C}{R} = 0$

$\omega = \omega_0 = \frac{1}{\sqrt{LC}} = 10^6$ rad/sec

i.e. $= \frac{v_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

for $\omega >> \omega_0$, circuit behaves as inductor



2. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following option is(are) correct?

(A) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

(B) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(C) For this prism, the emergent ray at the second surface will be the tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1}-\cos A\right]$

(D) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{z}\cos^{-1}\left(\frac{\mu}{2}\right)$

Ans. ABC

Sol. For mindeviation.

$$i = e$$

$r_1 + r_2 = A$

$\delta_m = 2i - A = A$ (Given)

$$r_1 = r_2 = \frac{A}{2} = \frac{i}{2}$$

(A) condition of min. deviation is i = e. & $r_1 = r_2 = \frac{A}{2}$

(C) $\mu = \frac{\sin(A)}{\sin(A/2)} \Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

This shows D is incorrect

$$\mu \sin r_2 = 1 \quad \Rightarrow \quad \sin r_2 = \frac{1}{\mu}$$

$$r_1 + r_2 = A \quad \Rightarrow \quad r_1 = A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$\sin i = \mu \sin r_1$

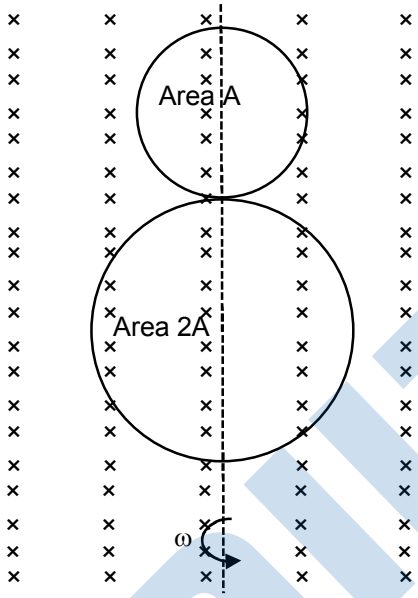
$$i = \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

putting $\mu = 2\cos\frac{A}{2}$

on solivng we get

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2}-1}-\cos A\right]$$

3. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(A) The emf induced in the loop is proportional to the sum of the areas of the two loops

(B) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(C) The net emf induced due to both the loops is proportional to cos ωt

(D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

Ans. BD

Sol. emf induced in coop 1 = $\frac{+d}{dt}$ (BA cos ωt)

$$2 = \frac{+d}{dt}(B\ 2A \cos \omega t)$$

$$\varepsilon_1 = + BA\ w.\ \sin(\omega t)$$

$$\varepsilon_2 = +B\ (2A)\ \omega \sin(\omega t)$$

so emf will be maximum when. $\omega t = \theta = \frac{\pi}{2}$

net emf = $\varepsilon_2 - \varepsilon_1 = 2B\omega \sin \omega t - BA \sin \omega t$

ε_{net} = BA $\omega \sin \omega t$ – BA ω sin ωt

e_{net} = BA ω sin ωt

4. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less then the average speed u of the gas molecules. Which of the following options is(are) true?

(A) At a later time the external force F balances the resistive force

(B) The plate will continue to move with constant non-zero acceleration, at all times

(C) The resistive force experienced by the plate is proportional tov

(D) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

Ans. ACD

Sol. [A] At terminal velocity external force balances resistance force.

[D] On collision with a heavy object the speed of the gas molecules resistance same.

On leading side :

Collision time = $T_l = \frac{2\ell}{u+v}$

On trailing side collision time $t_t = \frac{2\ell}{u-0}$

$F = \frac{\Delta p}{\Delta t}$

$F_{leading} = \frac{2mu}{2\ell}(u+v)$

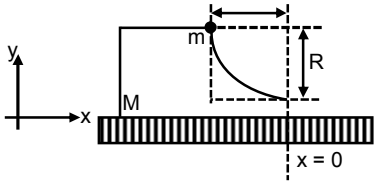
$F_{trailing} = \frac{2mu}{2\ell}(u-v)$

$\Delta_{pressure} = F_{leading} - F_{trailing}$

$= \frac{mu^2 + muv - mu^2 + muv}{\ell} = \frac{m}{\ell}uv$

$\therefore \Delta_{pressure} \propto uv$

5. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a coordinate system fixed to the table. A point of mass m is relased from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and velocity is v. At that instant, which of the following options is/are correct?



(A) The velocity of the point mass m is : $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

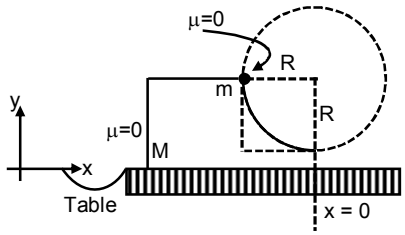
(B) The x component of displacement of the center of mass of the block M is :

(C) The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$

(D) The velocity of the block M is : $V = -\frac{m}{M}\sqrt{2gR}$

Ans. AB

Sol.



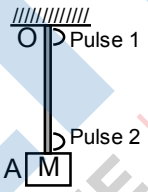
At that moment :(A) $v_m = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$?

(B) $v_M = \frac{-m}{M}\sqrt{2gR}$?

(C) $x_m = -\sqrt{2}\frac{mR}{M+m}$?

(D) x component of displacement of centre of mass of the block M is $\left(\frac{-mR}{M+m}\right)$

6. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



(A) The time $T_{AO}=T_{OA}$

(B) The wavelength of Pulse 1 becomes longer when it reaches point A

(C) The velocity of any pulse along the rope is independent of its frequency and wavelength

(D) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

Ans. AC

Sol. (A) speed of the wave is property of medium so time taken to cross the string will be equal

(C) velocity = $v = \sqrt{\frac{T}{\mu}}$

It depends upon tension and mass per unit length i.e. it is a property of medium hence independent from frequency and wave length

7. A human body has a surface area approximately $1m^2$. The normal body temperature is 10K above the surrounding room temperature T_0 . Take the room temperature to be T_0 = 300K. For T_0 = 300K, the value of σT_0^4 = 460 Wm^{-2} . (where σ is the Stefan Boltzmann content). Which of the following option is(are) correct?

(A) If the body temperature rises significantly then the peak in the spectrum of electromagnetic

(B) If the surrounding temperature reduced by a small amount $\Delta T_0 << T_0$, then to maintain the samebody temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy perunit time

(C) The amount of energy radiated by the body in 1 second is close to 60 Joules

(D) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation.]

Ans. BCD

Sol. (A) wein's law $\lambda_m T = b$ e = 1 (supposed)

i.e. If temperature increases that λ_m will be small so (A) is wrong

(B) $W_1 = \sigma A (T^4 - T_0^4)$

$W_2 = \sigma A (T^4 - (T_0 - \Delta T_0)^4)$

$W_2 - W_1 = 4\sigma A T_0^3 \Delta T_0$

$W_2 - W_1 = 4\sigma T_0^3 \Delta T_0$

SECTION 2 (Maximum Marks : 15)

- This section contains FIVE questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

8 An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f.. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is

Ans. 5

Sol. P.E. $\propto \frac{1}{n^2}$

$\frac{V_i}{V_F} = \frac{n_F^2}{n_i^2} = 6.25$

$n_F = 2.5\, n_i$

For minimum n_F

$n_F = 2.5 \times 2 = 5$ $(\therefore n_F \rightarrow \text{integer})$ $(n_i)_{minimum} = 2$

9. A drop of liquid of radius R = 10^{-2} m having surface tension S = $\frac{0.1}{4\pi}$Nm^{-1} divdes itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If K = 10^{α} then the value of α is

Ans. 6

Sol. R = 10^{-2} m

$S = \frac{1}{4\pi}Nm^{-1}$

$\frac{4}{3}\pi R^3 = k = \frac{4}{3}\pi r^3$ ($\therefore$ Volume is same)

$r = \frac{R}{k^{1/3}}$

$\Delta U = (k4\pi r^2 - 4\pi R^2)S$

$10^2 = (k^{1/3} - 1)$

$10^{\alpha/3} = 10^2 \Rightarrow \alpha = 6$

10. A stationary source emits sound of frequency f_0 = 492 Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting singal in Hz ? (Given that the speed of sound in air 330 ms^{-1} and the car reflects the sound at the frequency it has received).

Ans. 6

Sol. F_0 = 492 Hz

$F_{\text{received after reflection}} = 492\left(\frac{332}{330}\right)\left(\frac{330}{328}\right) = 498$

Beat frequency = 498 – 492 = 6 Hz

11. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4 × 10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 B The total volume of blood in the person's body, in liters in approximately (you may use $e^x \approx 1+x$ for $|x| << 1$ and $\ln 2 \approx 0.7$).

Ans. 5

Sol. $^{131}I \rightarrow Xe + B$

$t_{1/2}$ = 8 days = 8 × 24 hr

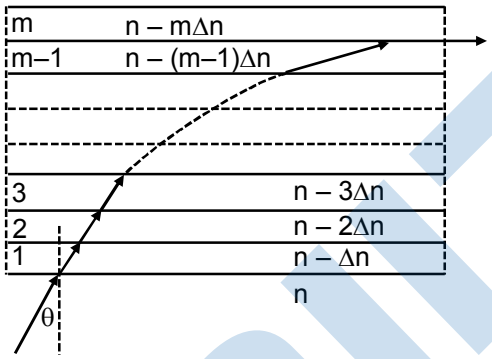
$h = \frac{\ell n 2}{8\times 24} = \frac{0.7}{8\times 24}$ $A = A_0 e^{-\lambda t}$

$A_{11.5}hr = A_0 e^{-\frac{(0.7)\times 11.5}{8\times 24}} = 1 - \frac{0.7\times 11.5}{8\times 24}$

$$115 \times \frac{V \times 1000}{2.5} \times (1 - 0.042) = 2.4 \times 10^5$$

$V = 5.4\ L = 5$

12. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interface of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m ?



Ans. 8

Sol. For parallel Refraction

$n \sin\theta = (n - m\Delta n) \sin 90°$

$m = 8$

SECTION 3 (Maximum Marks : 18)

- This section contains SIX questions of matching type.
- This section contains TWO tables (each having 3 columns and 4 rows)
- Based on each table, there are THREE questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONEof these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–1	In all other cases.

Answer 13, 14 and 15 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1	Column 2	Column 3

(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i)	$\vec{E} = E_0\hat{z}$	(P)	$\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii)	$\vec{E} = -E_0\hat{y}$	(Q)	$\vec{B} = B_0\hat{x}$
(III)	Proton with $\vec{v} = 0$	(iii)	$\vec{E} = -E_0\hat{x}$	(R)	$\vec{B} = B_0\hat{y}$
(IV)	Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv)	$\vec{E}$ $E_0\hat{x}$	(S)	$\vec{B} = B_0\hat{z}$

13. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$)?

(A) (IV) (ii) (S) (B) (II) (iii) (Q) (C) (III) (ii) (R) (D) (III) (ii) (P)

Ans. [C]

Sol. $\vec{F} = qvB = q(\vec{v} \times \vec{B})$

$\vec{F} = q\vec{E}$

14. In which case will the particle move in a straight line with constant velocity ?

(A) (II) (iii) (S) (B) (III) (iii) (P) (C) (IV) (i) (S) (D) (III) (ii) (R)

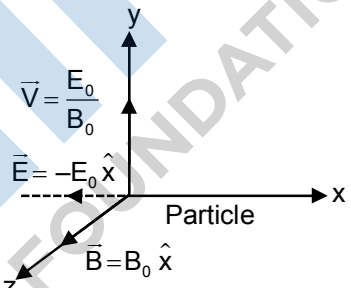
Ans. [A]

Sol. $F_{electro}$ $F_{magnetic}$

$F_{electro} = q \times E = -q\,E_0\,x$

$F_{net} = E_{Electro} + F_{magnetic} = 0$

Hence particle will move along straight line



15. In which case will the particle describe a helical path with axis along the positive z direction ?

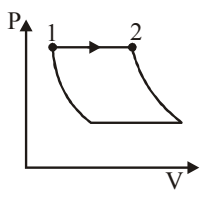
(A) (II) (ii) (R) (B) (III) (iii) (P) (C) (IV) (i) (S) (D) (IV) (ii) (R)

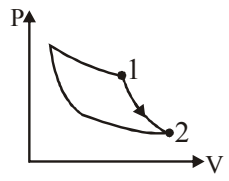
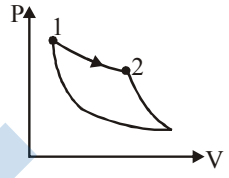
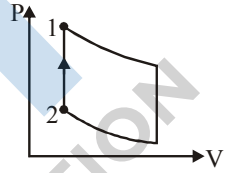
Ans. C

Sol. Force and velocity are perpendicular to each other.

Answer 16 to 18 by appropriately matching the information given in the three columns of the followingtable.

An ideal gas in undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P – V diagram in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas in n.

Column 1	**Column 2**	Column 3
(V) $W_{1\to2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$	(v) Isothermal	(P) 

(VI) $W_{1\to2} = -PV_2 + PV_1$	(vi) Isochoric	(Q) 
(VII) $W_{1\to2} = 0$	(vii) Isobaric	(R) 
(VIII) $W_{1\to2} = -nRT\ln(\frac{V_2}{V_1})$	(viii) Adiabatic	(S) 

16. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas ?

(A) (IV) (ii) (R) (B) (I) (ii) (Q) (C) (I) (iv) (Q) (D) (III) (iv) (R)

Ans. C

Sol. For determination of speed of sound in an ideal gas we use adianatic process. So any option - C is matching.

17. Which of the following options is the only correct representation of a process in which

$\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iii) (S) (B) (II) (iii) (P) (C) (III) (iii) (P) (D) (II) (iv) (R)

Ans. B

Sol. Only B option is following the correct sequence of operations.

18. Which one of the following options is the correct combination ?

(A) (II) (iv) (P) (B) (III) (ii) (S) (C) (II) (iv) (R) (D) (IV) (ii) (S)

Ans. B

Sol. 16 to 18

Isobaric $\rightarrow$ work done = $\rho\sigma v$

F.L.T. = Q = U + ω

$\Rightarrow$ W = adiabatic = $\frac{1}{r-1}(P_2v_2 - P_1v_1)$

$\Rightarrow W_{isochoric} = \rho dv = 0$

$\Rightarrow W_{isothermal} \Rightarrow -nRT \text{ long}\left(\frac{v_2}{v_1}\right)$

PART B: CHEMISTRY

SECTION 1 (Maximum Marks: 28)

- This section contains SEVEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three willget +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

19. The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to

(A) the physical state of X_2 at room temperature changes from gas to solid down the group

(B) decrease in HOMO-LUMO gap down the group

(C) decrease in $\pi^*-\sigma^*$ gap down the group

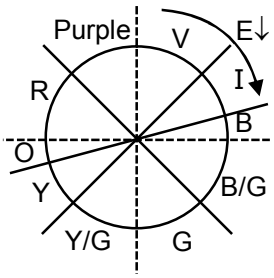
(D) decrease in ionization energy down the group

Ans. BC

Sol. F_2 (White) Cl_2 (Greenish yellow) Br_2 (Brown) I_2 (Violet)

* In Cl_2 large difference in between HOMO-LUMO or $\pi^*-\sigma^*$ gap it absorb high energy, so release less energy colour.

* In I_2 less difference in between HOMO-LUMO or $\pi^*-\sigma^*$ gap it absorb less energy colour, so release high energy violet colour.



20. Addition of excess aqueous ammonia to a pink coloured aqueous solution of $MCl_2.6H_2O$(X) and NH_4Cl gives an octahedral complex Y in the presence of air. In aqueous solution, complex Y behaves as 1 : 3 electrolyte. The reaction of X with excess HCl at room temperature results in the formation of a blue coloured complex Z. The calculated spin only magnetic moment of X and Z is 3.87 B.M., whereas it is zero for complex Y. Among the following options, which statement(s) is(are) correct?

(A) The hybridization of the central metal ion in Y is d^2sp^3

(B) When X and Z are in equilibrium at 0°C, the colour of the solution is pink

(C) Z is a tetrahedral complex

(D) Addition of silver nitrate to Y gives only two equivalents of silver chloride

Ans. ABC

Sol. $MCl_2.6H_2O + a\ NH_3 \xrightarrow{air} [M(NH_3)_6]Cl_3$ [1 : 3 electrolyte]

$[\mu_m = 3.87]$ [X] [Y]

Pink colour $[Co(NH_3)_6]Cl_3$

compound $\mu_m = 0$

↓ excess amount HCl

$[MCl_4]^{2-}$

[Z] Blue colour

$\mu_m = 3.87$

$[M(H_2O)_6]Cl_2 \rightleftharpoons [MCl_4]^{2-}$

if M is cobalt then

$[Co(H_2O)_6]Cl_2 \rightleftharpoons [COCl_4]^{2-}$ This is the example of invisible ink

[X]	[Z]
Pink colour	Blue colour
Pink colour	Blue colour

on upon heating

* Therefore at 0°C it is pink in colour

21. An ideal gas is expanded from (p_1, V_1, T_1) to (p_2, V_2, T_2) under different conditions. The correct statement(s) among the following is(are)

(A) If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic

(B) The work done by the gas is less when it is expanded reversibly from V_1 to V_2 under adiabatic conditions as compared to that when expanded reversibly from V_1 to V_2 under isothermal conditions

(C) The work done on the gas is maximum when it is compressed irreversibly from (p_2, V_2) to (p_1, V_1) against constant pressure p_1

(D) The change in internal energy of the gas is (i) zero, if it is expanded reversibly with $T_1 = T_2$, and (ii) positive, if it is expanded reversibly under adiabatic conditions with $T_1 \neq T_2$

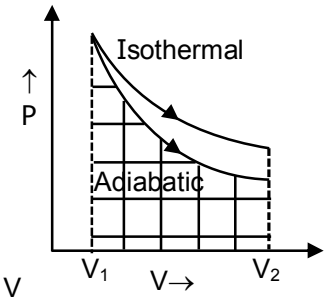
Ans. ABC

Sol. (A) For free expansion : w = 0 and q = 0

$\therefore \Delta U = q = 0$

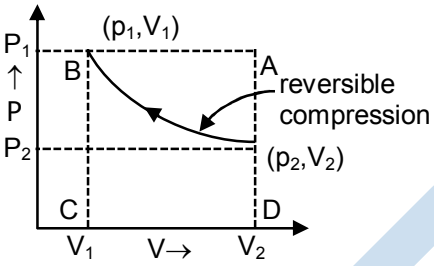
$\therefore$ The process is simultaneously both isothermal as well as adiabatic.

(B)



From above diagram it can be clearly noticed that area under adiabatic expansion curve is less than the area under isothermal curve.

(C)



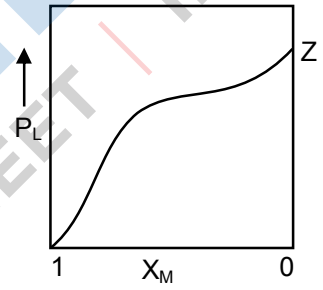
(D) (i) $\Delta U = nC_v\Delta T = nC_V (T_2 - T_1) = 0$ ($T_1 = T_2$ for isothermal)

(ii) $\Delta U = q + w$

During adiabatic expansion temperature will decreases

$\therefore$ Answer (ABC)

22. For a solution formed by mixing liquids L and M, the vapour pressure of L plotted against the mole fraction of M in solution is shown in the following figure. Here x_L and x_M represent mole fractions of L and M, respectively, in the solution. The correct statement(s) applicable to this system is (are)



(A) Attractive intermolecular interactions between L – L in pure liquid L and M–M in pure liquid M are stronger than those between L–M when mixed in solution

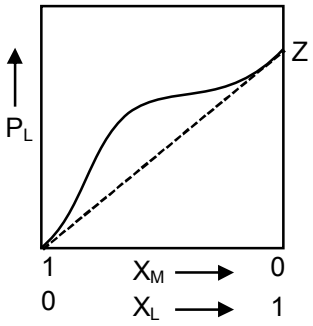
(B) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed when $x_L \rightarrow 0$.

(C) The point Z represents vapour pressure of pure liquid M and Raoult's law is obeyed from x_L=0 to $x_L = 1$

(D) The point Z represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \rightarrow 1$.

Ans. AD

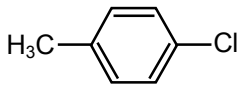
Sol. Solution is showing positive deviation.



The point Z represents vapour pressure of pure liquid L and Raoult's law is obeyed when $x_L \to 1$.

$\therefore$ Answer (AD)

23. The IUPAC name(s) of the following compound is(are)



(A) 1-chloro-4-methylbenzene

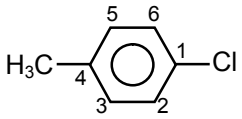
(B) 4-chlorotoluene

(C) 1-methyl-4-chlorobenzene

(D) 4-methylchlorobenzene

Ans. AB

Sol.



(A) 1-chloro-4-methylbenzene

(B) 4-chlorotoluene

24. The correct statement(s) for the following addition reactions is(are)

(A) O and P are identical molecules

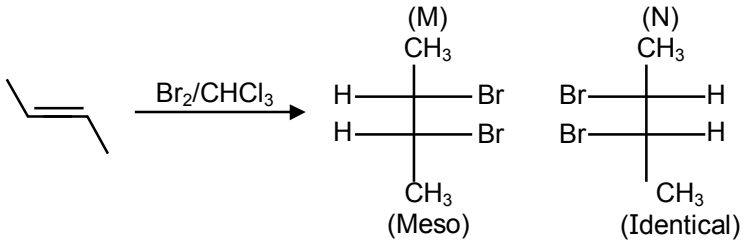
(B) Bromination proceeds through trans-addition in both the reactions

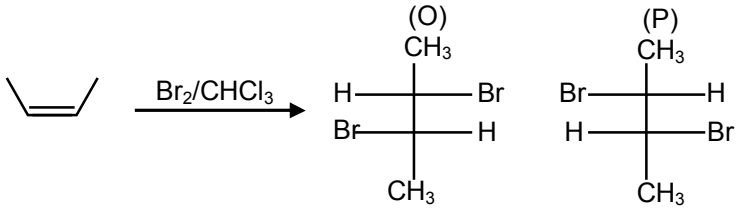
(C) (M and O) and (N and P) are two pairs of enantiomers

(D) (M and O) and (N and P) are two pairs of diastereomers

Ans. BD

Sol.





M and O → Diastereomers

N and P → Diastereomers

25. The correct statement(s) about the oxoacids, $HClO_4$ and HClO, is(are)

(A) The conjugate base of $HClO_4$ is weaker base than H_2O

(B) The central atom in both $HClO_4$ andHClO is sp^3 hybridized

(C) $HClO_4$ is formed in the reaction between Cl_2 and H_2O

(D) $HClO_4$ is more acidic than HClO because of the resonance stabilization of its anion

Ans. ABD

Sol. $HClO_4 \xrightarrow{-H^{\oplus}} ClO_4^{-}$

Strong acid Conjugate

sp^3hybridised base

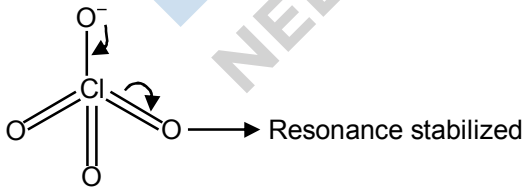
↓

(weak)

$H_2O \longrightarrow H^{\oplus} + OH^{-}$

W.A Conjugate base

Conjugate base of $HClO_4$ is a weak base while conjugate base of H_2O is strong



$HClO_3 \longrightarrow H^{\oplus} + ClO_3^{-}$

Acid Conjugate base

(sp^3hybridised)

$Cl_2 + H_2O \longrightarrow HOCl + HCl$

SECTION 2 (Maximum Marks : 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of following categories :

Full Marks	:	+3	If only the bubble corresponding to the correct answer is darkened.
Zero Marks	:	0	In all other cases.

26. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be 5×10^{-7} S. The pH of the solution is 4. The value of limiting molar conductivity $\left(\wedge_m^o\right)$ of this weak monobasic acid in aqueous solution is $Z \times 10^2$ S cm^{-1} mol^{-1}. The value of Z is

Ans. 6

Sol. $C = k\left(\frac{a}{\ell}\right)$

$\Rightarrow 5 \times 10^{-7}$ $\quad S = k \times \left(\frac{1cm^2}{120cm}\right)$

$\therefore k = 600 \times 10^{-7}$ S/cm

$\therefore \wedge_m = \frac{1000k}{M} = \frac{1000 \times 600 \times 10^{-7}}{15 \times 10^{-4}} Scm^2mol^{-1}$

$= 40$ S cm^2 mol^{-1}

$pH = 4 \Rightarrow [H^+] = 10^{-4}$

$\therefore$ $\quad H \times (aq) \quad \rightleftharpoons \quad H^+(a) + X^-(a)$

$t = t_{eq}$: $\quad (15 \times 10^{-4} - 10^{-4})M \quad 10^{-4}$ M $\quad 10^{-4}$ M

$\therefore \alpha = \frac{10^{-4}}{15 \times 10^{-4}} = \frac{\wedge_m}{\wedge_m^0}$

$\Rightarrow \frac{1}{15} = \frac{40}{\wedge_m^0}$

$\therefore 6 \times 10^2 Scm^2$ mol^{-1}

$\therefore$ Answer $\rightarrow 6$

27. The sum of the number of lone pairs of electrons on each central atom in the following species is $[TeBr_6]^{2-}$, $[BrF_2]^+$, SNF_3, and $[XeF_3]^-$

(Atomic numbers: N = 7, F = 9, S = 16, Br = 35, Te = 52, Xe = 54)

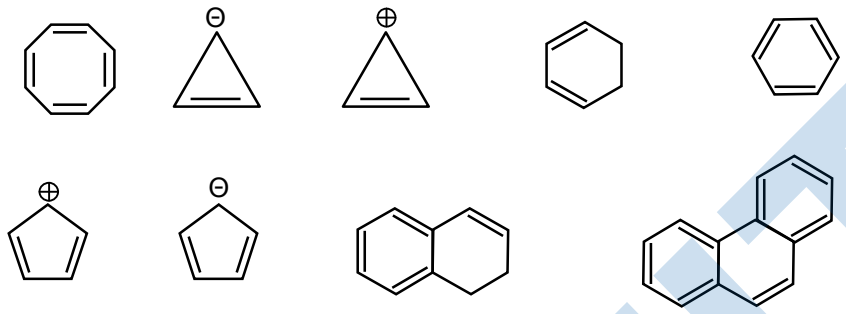
Ans. 6

Sol.

Compound	σ bonds + lone pair
$(TeBr_6)^{2-}$	6 + 1
$(BrF_2)^{+}$	2 + 2
SNF_3	4 + 0
$(XeF_3)^{-}$	3 + 3

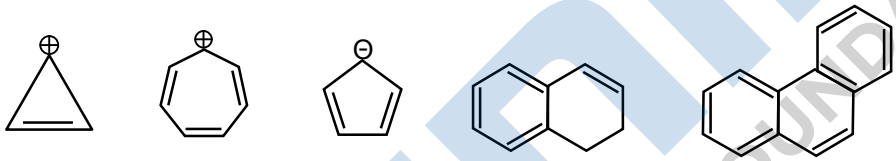
Total lone pair of electrons = 1 + 2 + 0 + 3 = 6

28. Among the following, the number of aromatic compound(s) is



Ans. 5

Sol.



29. A crystalline solid of a pure substance has a face-centred cubic structure with a cell edge of 400 pm. If the density of the substance in the crystal is 8 g cm^{-3}, then the number of atoms present in 256 g of the crystal is N × 10^{24}. The value of N is

Ans. 2

Sol. $d = \frac{ZM}{N_A.a^3}$

$\Rightarrow 8 = \frac{4 \times M}{(6.022 \times 10^{23}) \times (4 \times 10^{-8})^3}$ = 128 × 0.6022 g / mol

$n = \frac{256}{128 \times 0.6022} mol = \left(\frac{2}{0.6022}\right) mol$

$\therefore$ Number of atoms = $\frac{2}{0.6022} \times 6.022 \times 10^{23}$

= 2 × 10^{24}

$\therefore$ Answer $\rightarrow$ 2

30. Among H_2, He_2^+, Li_2, Be_2, B_2, C_2, N_2, O_2^-, and F_2, the number of diamagnetic species is (Atomic numbers : H = 1, He = 2, Li = 3, Be = 4, B = 5, C = 6, N = 7, O = 8, F = 9)

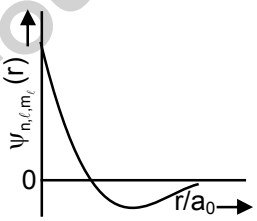
Ans. 6

Sol.

	U.P.E.	Paramagnetic / Diamagnetic
H_2	0	Diamagnetic
He_2^+	1	Paramagnetic
Li_2	0	Diamagnetic
Be_2	0	Diamagnetic
B_2	2	Paramagnetic
C_2	0	Diamagnetic
N_2	0	Diamagnetic
O_2^-	1	Paramagnetic
F_2	0	Diamagnetic

Answer 31, 32 and 33 by appropriately matching the information given in the three columns of the following table.

The wave function $\Psi_{n.1,m_\ell}$ is a mathematical function whose value depends upon spherical polar coordinates (r,ω,ϕ) of the electron and characterized by the quantum numbers n,ℓ and m_ℓ. Here r is distance from nucleus, θ is colatitude and ϕ is azimuth. In the mathematical functions given in the Table, Z is atomic number and a_0 is Bohr radius.

Column 1	Column 2	Column 3
(i) 1 s orbital	(i) $\Psi_{n,\ell,m_\ell} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\left(\frac{Zr}{a_0}\right)}$	(P) 
(ii) 2s orbital	(ii) One radial node	(Q) Probability density at nuvleus $\propto \frac{1}{a_0^3}$
(iii) $2p_z$ orbital	(iii) $\Psi \propto \left(\frac{Z}{a_0}\right)^{\frac{5}{2}} re^{-\left(\frac{Zr}{za_0}\right)_{\cos\theta}}$	(R) Probability density is maximun at nucleus
(iv) $3d_{Z^2}$ orbital	(iv) xy- plane is a nodal plane	(S)Energy needed to excite electron from n=2 state to n=4 state is times the energy needed to excite electron from n=2 state

3. For He^+ ion, the only INCORRECT combination is

(A) (I) (i) (R) (B) (II) (ii) (Q) (C) (I) (i) (S) (D) (I) (iii) (R)

Ans. D

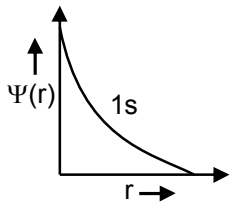
Sol. For He^+ ion

(A) For 1s-orbital, $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} .e^{-\left(\frac{Zr}{a_0}\right)}$

and probability density = $\left|\psi_{n,l,m_l}\right|^2$

s-orbital has no angular node.

$\therefore$ (I) (i) (R) is correct combination



(B) For 2s-orbital. There us one radial node & $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} .e^{-\left(\frac{Zr}{2a_0}\right)}$

$\therefore$ Probability density $\propto \frac{1}{a_0^3}$

$\therefore$ (II) (ii) (Q) is correct combination

(C) For 1s-orbital : $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} .e^{-\left(\frac{Zr}{a_0}\right)}$

$(\Delta E_{2\to4})_{He^+} = (-0.85 + 3.4) \times 2^2$ eV

$(\Delta E_{2\to4})_{He^+} = 13.6 \times \left(\frac{1}{4} - \frac{1}{36}\right) \times 2^2 eV = (2.55 \times 2^2) eV$

$= 3.022 \times 2^2$ eV

$\Rightarrow \frac{27}{32} \times 3.022 \times 2^2 = 2.55 \times 2^2$

$\therefore$ (I) (i) (S) is correct combination

(D) For 1s-orbital : $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} .e^{-\left(\frac{Zr}{a_0}\right)}$

$\therefore$ (I) (iii) are incorrect combination

$\therefore$ Answer (D)

32. For the given orbital in column 1, the only CORRECT combination for any hydrogen-like species is

(A) (I) (ii) (S) (B) (IV) (iv) (R) (C) (III) (iii) (P) (D) (II) (ii) (P)

Ans. D

Sol. For 2s-orbital :

Number of radial node = 1

& probability density at nucleus = $\left|\psi_{n,l,m_l}\right|^2$

i.e. probability density at nucleus $\propto \frac{1}{a_0^3}$

$\therefore$ (II) (ii) (P) is incorrect combination

$\therefore$ Answer (D)

33. For hydrogen atom, the only CORRECT combination is

(A) (II) (i) (Q) (B) (I) (iv) (R) (C) (I) (i) (P) (D) (I) (i) (S)

Ans. D

Sol. (D) For 1s-orbital : $\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} .e^{-\left(\frac{Zr}{a_0}\right)}$

$(\Delta E_{2\to4})_H$= (–0.85 + 3.4) eV = 2.55 eV

$(\Delta E_{2\to6})_H = 13.6\left(\frac{1}{4}-\frac{1}{36}\right)$eV = 3.022

But, $3.022 \times \frac{27}{32} = 2.55$

∴ (I) (i) (S) is correct combination

∴ Answer (D)

Answer 34, 35 and 36 by appropriately matching the information given in the three columns of the following table.

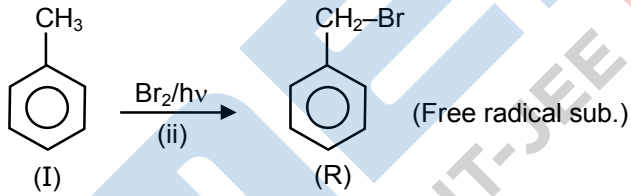
Column 1,3 and 3 contain starting materials, reaction conditions, and type of reactions, respectively.

Column1	**Column 2**	**Column 3**
(I) Toluene	(II) NaOH / Br_2	(P) Condensation
(II) Acetophenone	(II) $Br_2/h\nu$	(Q) Carboxylation
(III) Benzaldehyde	(III) $(CH_3CO)_2O/CH_3COOK$	(R) Substitution
(iv) Phenol	(iv) $NaOH/CO_2$	(S) Haloform

34. The only CORRECT combination in which the reaction proceeds through radical mechanism is

(A) (II) (iii) (R) (B) (III) (ii) (P) (C) (IV) (i) (Q) (D) (I) (ii) (R)

Ans. D

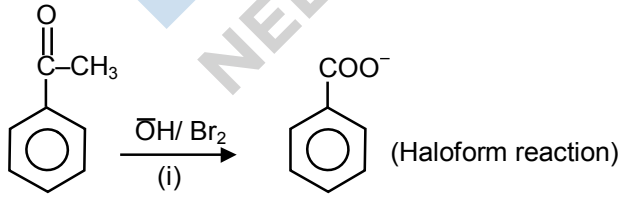
Sol. 

(I) (R) (Free radical sub.)

35. For the synthesis of benzoic acid, the only CORRECT combination is

(A) (III) (iv) (R) (B) (IV) (ii) (P) (C) (II) (i) (S) (D) (I) (iv) (Q)

Ans. C

Sol. 

(II) (S) (Haloform reaction)

36. The only CORRECT combination that gives two different carboxylic acids is

(A) (IV) (iii) (Q) (B) (I) (i) (S) (C) (III) (iii) (P) (D) (II) (iv) (R)

Ans. C

Sol. $\underset{(III)}{Ph-CHO} \xrightarrow[\text{Perkin condensation}]{(CH_3CO)_2O/CH_3COOK} Ph-CH=CH-COOH$ (C / T)

PART C: MATHEMATICS

SECTION 1 (Maximum Marks : 28)

- This section contains SEVEN questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four options is(are) correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

37. Let a, b, x and y be real numbers such that a – b = 1 and $y \neq 0$. If the complex number z = x + iy satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x?

(A) $-1+\sqrt{1-y^2}$ (B) $1-\sqrt{1+y^2}$ (C) $1+\sqrt{1+y^2}$ (D) $-4-\sqrt{1-y^2}$

Ans. AD

Sol. $a - b = 1; \quad y \neq 0, a, b, c, y \in R$

$z = x + iy$

$$\frac{az+b}{z+1}=\frac{ax+aiy+b}{x+iy+1}=\frac{((ax+b)+iay)((x+1)-iy)}{(x+1)^2+y^2}$$

$$\text{Im}\left(\frac{az+b}{z+1}\right)=\frac{ay(x+1)-(ax+b)y}{(x+1)^2+y^2}=\frac{ay-by}{(x+1)^2+y^2}=y$$

$\therefore \quad (x+1)^2 + y^2 = 1 \quad (\because a - b = 1)$

$\Rightarrow \quad x + 1 = \pm\sqrt{1-y^2} \quad \Rightarrow \quad x=-1+\sqrt{1-y^2}$ and $-1-\sqrt{1-y^2}$.

38. Let $f : R \to (0, 1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

(A) $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t\,dt$ (B) $x^9 - f(x)$

(C) $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$ (D) $e^x - \int_0^x f(t)\sin t\,dt$

Ans. BC

Sol $f : R \to (0, 1)$

(A) Let $\psi(x) = f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt$

$\psi(0) = f(0) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt > 0$

$\psi(1) = f(1) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt > 0$

$\therefore$ no root

(B) Let $h(x) = x^9 - f(x)$

$h(0) = -f(0) < 0$

and $h(1) = 1 - f(1) > 0$

(C) Let $g(x) = x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$

$g(0) = 0 - \int_0^{\frac{\pi}{2}} f(t) \cos t \, dt < 0; g(1) = 1 - \int_0^{\frac{\pi}{2}} f(t) \cos t \, dt > 0$

$\therefore$ $g(x) = 0$ foratleast one $x \in (0, 1)$

(D) Let $\phi(x) = e^x - \int_0^x f(t) \sin t \, dt$

$\phi(0) = 1 - \int_0^1 f(t) \sin t \, dt > 0 ; \phi(1) = e - \int_0^e f(t) \sin t \, dt > 0.$

39. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle?

(A) 2a, 4, 1 (B) a, 4, 1 (C) a, 4, 2 (D) 2a, 8, 1

Ans. BCD

Sol. $2x - y + 1 = 0$

$\frac{x^2}{a^2} - \frac{y^2}{16} = 1$

$y = mx \pm \sqrt{a^2m^2 - 16}$ .....(1)

$y = 2x + 1$ .....(2)

$m = 2$ and $a^2m^2 - 16 = 1$

$4a^2 = 17 \quad \Rightarrow \quad a^2 = \frac{17}{4}$

$\therefore \quad (2a)^2 = 4^2 + 12$

$\therefore$ 2a, 4, 1 is right angled triangle.

40. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X \mid Y) = \frac{1}{2}$ and $P(Y \mid X) = \frac{2}{5}$. Then

(A) $P(X \cap Y) = \frac{1}{5}$ (B) $P(Y) = \frac{4}{15}$ (C) $P(X'|Y) = \frac{1}{2}$ (D) $P(X \cup Y) = \frac{2}{5}$

Ans. BC

Sol. $P(X) = \frac{1}{3}$; $\frac{P(Y \cap X)}{P(X)} = \frac{2}{5} \Rightarrow P(X \cap Y) = \frac{2}{15}$

$P(X|Y) = \frac{1}{2} \Rightarrow \frac{2/15}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{4}{15} \Rightarrow$ (B)

$P(X'|Y) = \frac{P(X' \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)} = \frac{\frac{4}{15} - \frac{2}{15}}{\frac{4}{15}} = \frac{1}{2} \Rightarrow$ (C)

$P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$.

41. Which of the following is(are) NOT the square of a 3 × 3 matrix with real entries?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Ans. AC

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

42. Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x \cos(\pi(x+[x]))$ is discontinuous?

(A) x = 0 (B) x = 1 (C) x = 2 (D) x = – 1

Ans. BCD

Sol. $f(x) = x \cos\left(\pi(x+[x])\right) = \begin{cases} x\cos(\pi(x+n)), & n \le x < (n+1) \\ x\cos(\pi(x+(n-1))), & n \le x < (n+1) \end{cases}$

L.H.L. (at x = n) = n cos $(2n-1)\pi$)= – n

R.H.L. (at x = n) = n cos$(2x\pi)$ = n

F(x) = n

for cont. L.H.L. = R.H.L. = f (x) $\Rightarrow$ n = 0

$\therefore$ f (x) is discont. at x = – 1, 1, 2

.

43. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k?

(A) p = 2, h = 3, k = – 4 (B) p = 5, h = 4, k = – 3

(C) p = – 1, h = 1, k = – 3 (D) p = – 2, h = 2, k = – 4

Ans. A

Sol. $y^2 = 16x$

Chord is $2x + y = p \Rightarrow y = -2x + p$ .....(1)

Chord with mid point (h, k)

$T = S_1 \Rightarrow yk - 8(x + h) = k^2 - 16h$

$$y = \frac{8x}{k} + \frac{k^2 - 8h}{k}$$ .....(2)

(1) & (2) represent same line.

$$\therefore \frac{8}{k} = -2 \Rightarrow k = -4. \text{ and } p = \frac{k^2 - 8h}{k} = \frac{16 - 8h}{-4}$$

$p = -4 + 2h$

h = 2, p = 0 and h = 3, p = 2.

SECTION 2 (Maximum Marks : 15)

- This section contains FIVE questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

44. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

Ans. 6

Sol. Let sides a, b, c with $a < b < c$

$2b = a + c$

$\therefore c^2 = a^2 + b^2$

$$\text{Area} = \frac{1}{2}ab = 24 \Rightarrow ab = 48.$$

$\therefore a^2 + b^2 = (2b - a)^2 = 4b^2 + a^2 - 4ab$

$\Rightarrow 3b^2 = 4 \times 48 \Rightarrow b^2 = 64 \Rightarrow b = 8.$

$\therefore a = 6.$

45. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Ans. 1

Sol. For infinite solution $|A| = 0$ and (adj A) B = 0

$$\Rightarrow |A| = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$\begin{vmatrix} -(\alpha-1) & -\alpha(\alpha-1) & \alpha^2 \\ (\alpha-1) & -(\alpha-1) & \alpha \\ \alpha(\alpha-1) & (\alpha-1) & 1 \end{vmatrix} = 0$$

$$\Rightarrow (\alpha-1)^2 \begin{vmatrix} -1 & -\alpha & \alpha^2 \\ 1 & -1 & \alpha \\ \alpha & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (\alpha-1)^2 \{-1(-1-\alpha) + \alpha(1-\alpha^2) + \alpha^2(1+\alpha)\} = 0$$

$$\Rightarrow (\alpha-1)^2 \{\alpha^2 + 2\alpha + 1\} = 0$$

$\Rightarrow \alpha = 1$ or $\alpha = -1$

(i) If $\alpha = 1$, then plane will be parallel

(ii) If $\alpha = -1$, then $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ (adj A) B = 0

$\therefore \alpha^2 + \alpha + 1 = 1.$

46 Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

Ans. 5

Sol. $x = 10!$ and $y = {}^{10}C_9 \times {}^{9}C_1 \times \frac{10!}{2!} = 5 \times 9 \times 10!$

$\therefore \frac{y}{9x} = 5.$

47. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

Ans. 2

Sol.

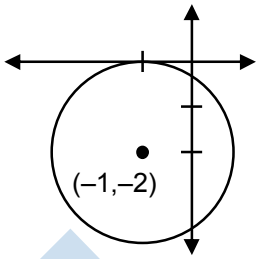
(i) Circle touches x-axis and intersect y-axis at two distinct points.

$\because g^2 = C \Rightarrow 1 = -p \Rightarrow p = -1$

and $f^2 - C > 0 \Rightarrow 4 - p > 0$

$\Rightarrow p < 4$

$\therefore p = -1$



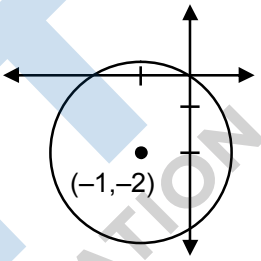
(ii) Circle touches y-axis and intersect x-axis

$f^2 = C \Rightarrow p = 4$

and $g^2 - C > 0 \Rightarrow 1 - p > 0$

$\Rightarrow p < 1$

$\therefore$ No value of p.



(iii) Circle passes through origin.

$\therefore p = 0$

$\therefore$ Number of values of p = 2.

48. Let $f : R \to R$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$.

If $g(x) = \int_x^{\frac{\pi}{2}} \left[f'(t)\,\text{cosec}\,t - \cot t\,\text{cosec}\,t\,f(t)\right] dt$ for $x \in \left(0, \frac{\pi}{2}\right]$ then $\lim_{x\to 0} g(x) =$

Ans. 2

Sol. $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$

$$g(x) = \int_x^{\frac{\pi}{2}} \left[\underbrace{f'(t)}_{II}\underbrace{\text{cosec}\,t}_{I} - \cot t\,\text{cosec}\,t\,f(t)\right] dt$$

$$g(x) = \left(\text{cosec}\,t\,f(t)\right)_x^{\frac{\pi}{2}} + \int_x^{\frac{\pi}{2}} \text{cosec}\,t\cot t\,f(t)\,dt - \int_x^{\frac{\pi}{2}} \cot t\,\text{cosec}\,t\,f(t)\,dt$$

$$= f\left(\frac{\pi}{2}\right)\cdot 1 - f(x)\,\text{cosec}\,x = 3 - \frac{f(x)}{\sin x}$$

$$\therefore \lim_{x\to 0} g(x) = 3 - \lim_{x\to 0}\frac{f(x)}{x} = 3 - f'(0) = 2$$

SECTION 3 (Maximum Marks : 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are THREE questions.
- Each question has **FOUR** options (A), (B), (C) and (D). ONLY ONEof these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–1	In all other cases.

Answer 49, 50 and 51 by appropriately matching the information given in the three columns of the following table.

Colum 1,2 and 3 contain conics, equation of tangents to the conics and points of contact, respecitively.		
(i) $x^2+y^2=a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(ii) $x^2+ a^2y^2= a^2$	(ii) $u = mx + a$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(ii) $y^2= 4ax$	(iii) $y = mx + a\sqrt{m^2+1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(iv) $x^2– a^2y^2= a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination?

(A) (IV) (iii) (S) (B) (II) (iii) (R) (C) (IV) (iv) (S) (*D) (II) (iv) (R)

50. If a tangent to a suitable conic (Column 1) is found to be y = x + 8 and its point of contact is (8, 16), then which of the following options is the only CORRECT combination?

(*A) (III) (i) (P) (B) (I) (ii) (Q) (C) (II) (iv) (R) (D) (III) (ii) (Q)

51. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (– 1, 1), then which of the following options is the only CORRECT combination for obtaining its equation?

(A) (II) (ii) (Q) (B) (I) (i) (P) (*C) (I) (ii) (Q) (D) (III) (i) (P)

Ans.

Sol. (49,50,51)

(i) $x^2 + a^2y^2 = a^2$

$3 + \frac{a^2}{4} = a^2 \Rightarrow b = \frac{3a^2}{4} \Rightarrow^2 = 4 \quad \Rightarrow \quad a = 2$

$x^2 + 4y^2 = 4$

$\frac{x^2}{4} + \frac{y^2}{1} = 1$

(ii) $y^2 = 4ax \Rightarrow (16)^2 = 4a \cdot 8 \Rightarrow a = 8$

(iii) for $a = \sqrt{2}$, $(-1, 1)$ is lying on the circle $x^2 + y^2 = 2$.

Answer 52, 53 and 54 by appropriately matching the information given in the three columns of the following table.

Let $f(x) = x + \ell og_e x - x \ell og_e x$, $x \in (0,\infty)$.

- Column 1 contains information about zeros of f(x), f'(x) and f"(x).
- Column 2 contains information about the limiting behavior of f(x), f'(x) and f" (x) at infinity
- Column 3 contain information about increasing/decreasing nature of f(x) and f'(x).

Column 1	**Column 2**	**Column 3**
(i) f(x) = 0 for some $x \in (1, e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in (0,1)
(ii) f'(x) = 0 for some $x \in (1, e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(iii) f'(x) = 0 for some $x \in (0,1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0,1),
(iv) f"(x) = 0 for some$x \in (1, e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e,e^2)

52. Which of the following options is the only CORRECT combination?

(A) (I) (ii) (R) (B) (IV) (i) (S) (C) (III) (iv) (P) (*D) (II) (iii) (S)

53. Which of the following options is the only CORRECT combination?

(A) (I) (i) (P) (*B) (II) (ii) (Q) (C) (III)(iii) (R) (D) (IV) (iv) (S)

54. Which of the following options is the only INCORRECT combination?

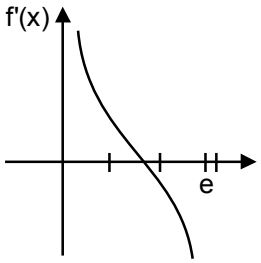
(A) (II) (iii) (P) (B) (I) (iii) (P) (*C) (iii) (i) (R) (D) (II) (iv) (Q)

Sol. (52,53,54)

$f(x) = x + \ln x - x \ln x$

$f(1) = f(e) = 1$

$f'(x) = 1 + \frac{1}{x} - x \cdot \frac{1}{x} - \ell n\, x = \frac{1}{x} - \ell n\, x$

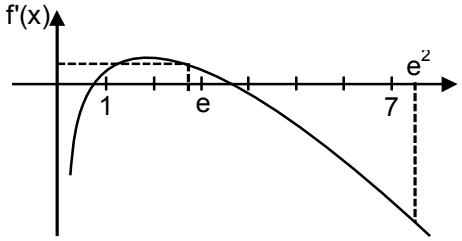


$f''(x) = \frac{-1}{x^2} - \frac{1}{x} = \frac{-(1+x)}{x^2}$

$f''(x) < 0 \quad \forall x \in (0, \infty) \Rightarrow f'(x)$ is $\downarrow$

$f'(0^+) \to \infty, f'(\infty) \to -\infty$

$f'(1) = 1$



$f'(e) = \frac{1}{e} - 1 < 0$

Now verify.